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COMPARISON OF SINUSOIDAL PULSEWIDTH – MODULATION METHODS

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Abstract

The different sinusoidal (natural) sampling methods for bipolar and unipolar pulsewidth modulation of one- and three-phase inverters are compared. The load voltage spectra, the voltage, the flux and the current distortion factors are determined, and on this basis it is shown that unipolar modulation produces a lower value of harmonic current losses than bipolar modulation does, particularly, for one-phase inverters and high value of fundamental voltage.

Keywords: PWM, inverter, distortion factors, two-level and three-level inverters.

I. Introduction

Sinusoidal natural sampling methods [1–3] are widely used for pulsewidth modulation (PWM) control of one- and three-phase inverters. For the one-phase inverter in *Fig. 1.a*, the PWM strategy in *Fig. 2.a* can be used; here the u_L load voltage is determined by the intersections of the carrier triangular wave and the reference sinusoidal signal. If the triangular wave is higher than the sinusoidal one, the T2 transistor is turned on ($u_L < 0$); in the opposite case the T1 transistor is turned on ($u_L > 0$). Hence, u_L can be equal to $U_{dc}/2$ or $-U_{dc}/2$, which creates the so-called bipolar modulation.

The strategy in *Fig. 2.a* can also be used for *Fig. 1.b*, but better results can be obtained by using the unipolar modulation methods in *Fig. 2.b* and *Fig. 2.c*. In *Fig. 2.b* the carrier wave is shifted on the abscissa axis; therefore, between $0 \leq W_1 t \leq \pi$, u_L load voltage can be positive or zero (if the triangular wave exceeds the sinusoidal one), and negative or zero for $\pi < W_1 t \leq 2\pi$. In *Fig. 2.c* the middle point of the pulses is shifted by T cycle time, but the narrow of the pulses is proportional to the value of $A \sin W_1 t$ in the middle point of the pulses. The last two PWM methods produce unipolar modulation of u_L voltage.

All the PWM methods create load voltage harmonics, which produce current harmonics and unwanted harmonic additional losses. If the load

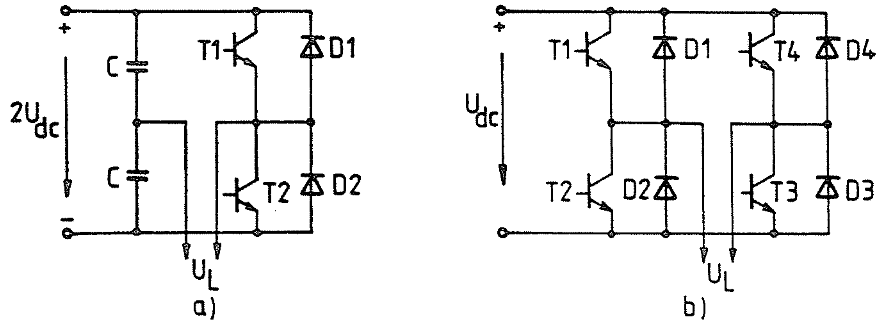


Fig. 1. One-phase inverters

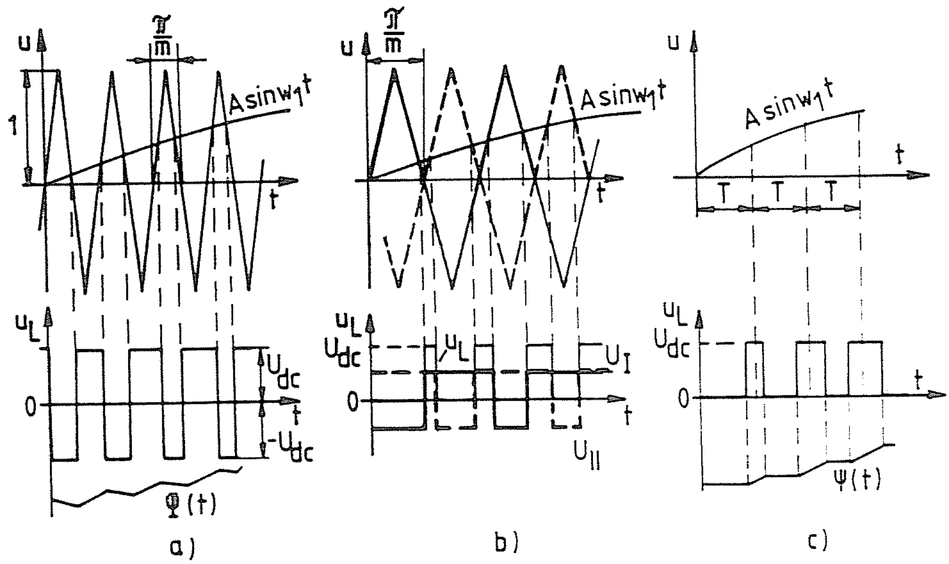


Fig. 2. PWM methods
a) bipolar, natural sampling
b) unipolar, natural sampling and
c) unipolar, regular sampling

voltage is sinusoidal, the voltage harmonics must be filtered by appropriate LC circuits. In the case of a motor load, a more reasonable solution is to decrease the current harmonics by increasing the carrier wave frequency. The harmonic losses can be minimized, for a given value of the fundamental voltage, by optimizing the switching times of the transistors as well [4–6].

But realization of the optimal PWM strategy is more difficult than natural sampling, especially, in the case of high commutation frequency of the inverter and for on-line realization of PWM.

The different PWM methods can be compared by examining the load voltage spectra and values of the voltage and current distortion factors.

2. One–phase Inverters

A) Voltage Spectra

The voltage spectrum of the load voltage according to *Fig. 2.a* can be written as follows [7–8]:

$$u_L = A \cdot U_{dc} \cdot \sin(W_1 \cdot t + \varphi) + \sum_{\nu \neq 1}^{\infty} U_\nu \cdot \sin(\nu \cdot W_1 \cdot t + n \cdot \varphi), \qquad (1)$$

where

$$U_\nu = \frac{2}{\pi \cdot K} \cdot U_{dc} \cdot J_n \cdot \left(K \cdot A \cdot \frac{\pi}{2}\right) \cdot \left[(-1)^n - (1)^k\right], \qquad (2)$$

and

$\nu = \pm Km \pm n > 0$ — the order of harmonics
W_1 — angular velocity of the fundamental component
t — time
A — amplitude of the reference wave ($A \leq 1$)
K — positive integer
n — positive integer or zero
J_n — first-kind Bessel function of the n order
m — ratio of the frequencies of the carrier and reference waves
φ — angle between the carrier and reference waves (in *Fig. 2* $\varphi = 0$).

Harmonics with a given order can be obtained by different pairs of K and n, but for $m > 6$ only the harmonics belonging to the smallest value of n can be of any importance. As it is well known, the order of the important harmonics is as follows: m, $m \pm 2$, $3m$, $3m \pm 2$, $3m \pm 4$ and $2m \pm 1$, $2m \pm 3$, $4m \pm 1$, $4m \pm 3$, $4m \pm 5$. The relative amplitudes of the harmonics are presented in *Fig. 3* and *Fig. 4*. In *Fig. 3*, the relative amplitudes U_ν/U_{dc} are gathered as function of A, those harmonics with $n = 3$ which have a high value and fundamental as well. One can see that the fundamental voltage is proportional to A with a very good approximation.

In *Fig. 4* the U_ν/U_1 relative amplitudes are drawn as a function of U_1/U_{dc}.

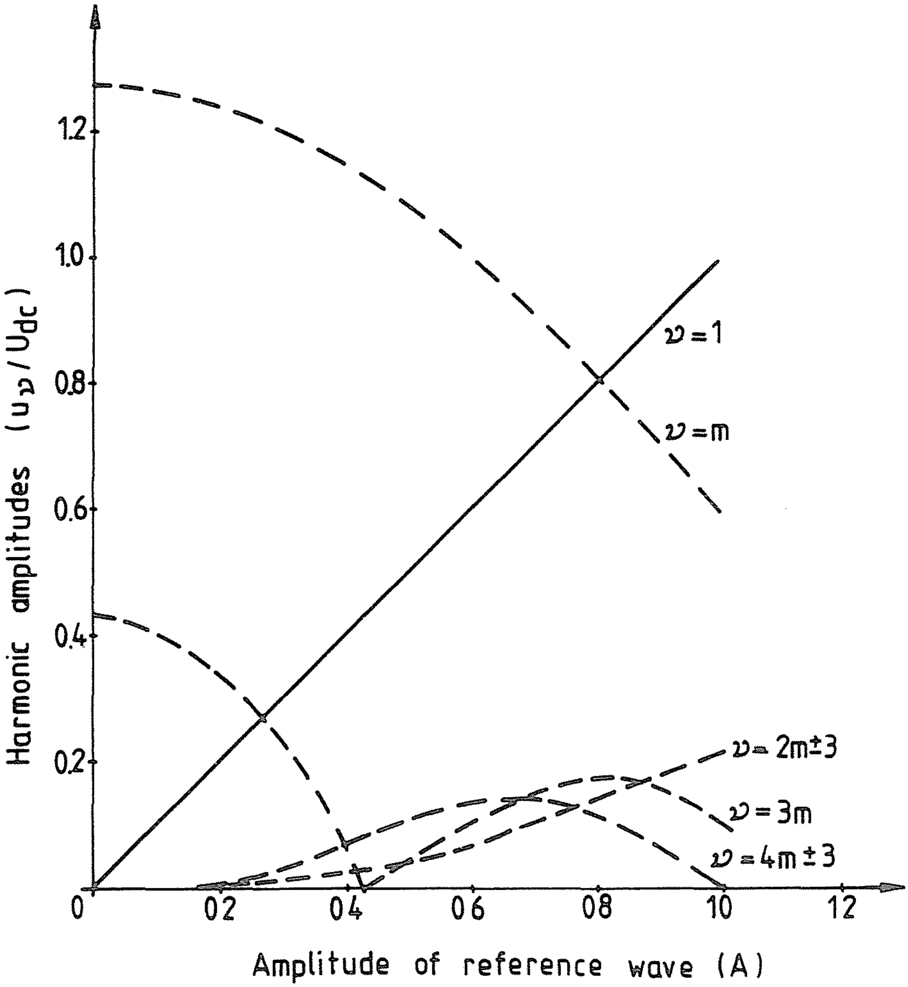


Fig. 3. Fundamental voltage and voltage harmonics for $n = 0$ and $n = 3$

It can be shown that the u_L voltage-time function for the unipolar modulation of *Fig. 2.b* can be composed by the sum of the two bipolar ones [9], as shown in *Fig. 2.b*. If we put the origin in the time, where the sinusoidal function is equal to zero, then the phase angle between the reference and first carrier waves will be $\varphi = 0$; and between reference and second carrier wave, $\varphi + \pi/m$. Using this and taking into account (2), U_1

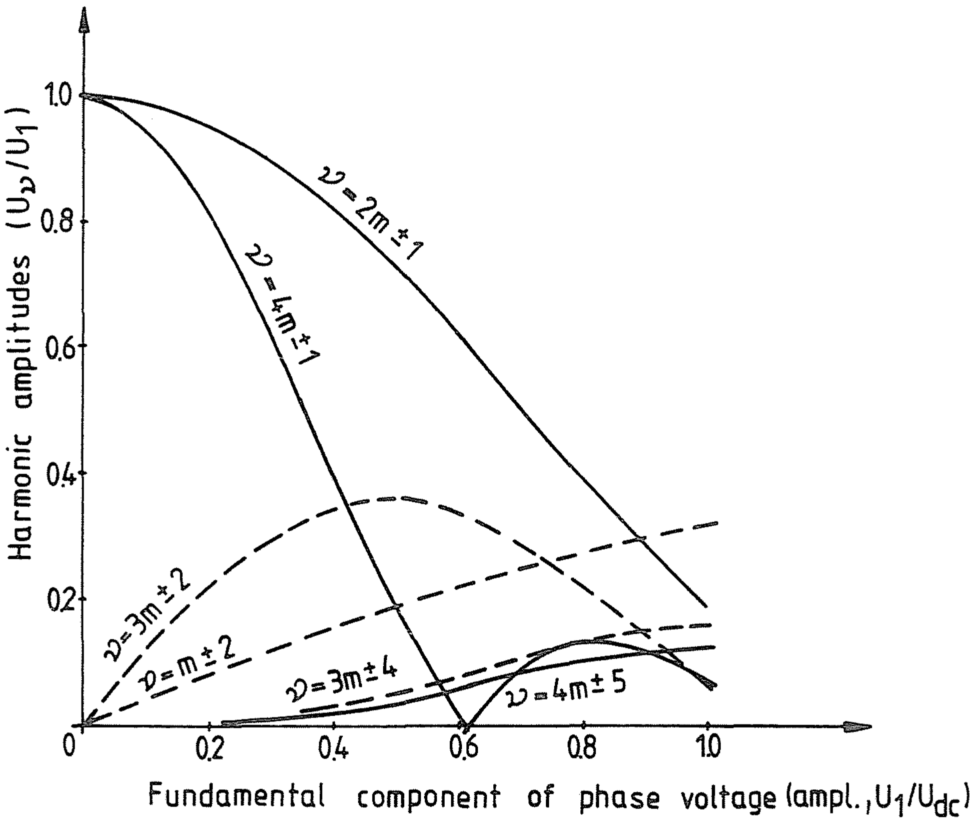


Fig. 4. Important voltage harmonics as a function of the fundamental voltage

and U_{II} the voltage-time function can be written in the following form:

$$U_{\mathrm{I}} = A\frac{U_{dc}}{2}\sin W_1 t + \sum_{\nu>1}\frac{U_\nu}{2}\sin(\nu W_1 t - Km\varphi),$$
$$U_{\mathrm{II}} = A\frac{U_{dc}}{2}\sin W_1 t + \sum_{\nu>1}\frac{U_\nu}{2}\sin[\nu W_1 t - Km(\varphi + \pi/m)]. \qquad (3)$$

The load voltage is the sum of U_{I} and U_{II}:

$$u_L = AU_{dc}\sin W_1 t + \sum_{\nu>1} U_\nu \sin(\nu W_1 t - Km\varphi), \qquad (4)$$

where $K = 2, 4, 6 \ldots$ and the U_ν harmonic amplitudes are determined by (3). Consequently, the harmonics for $K = 1, 3, 5$ (harmonics with the dotted lines in *Fig. 3* and *Fig. 4*) are eliminated. Hence, the order of the important harmonics becomes $2m \pm 1$, $2m \pm 3$, $4m \pm 1$, $4m \pm 3$, $4m \pm 5$, $6m \pm 1$.

The modulation process in *Fig. 2.c* produces regular sampling [10]. The amplitudes of the voltage spectrum are expressed by:

$$U_\nu = \frac{4m}{\nu\pi} J_n \left(A\nu \frac{\pi}{2m} \right), \tag{5}$$

where $\nu = Km + n > 0$ and $K = 2, 4, 6, \ldots$.

Practically, for a high value of m, the amplitudes and the order of harmonics for modulation processes in *Fig. 2.b* and *Fig. 2.c* will be the same.

B) Voltage and Current Distortion Factors

The quality of PWM should be determined by the voltage distortion factor

$$K_U = \frac{2U_L^2 - U_1^2}{U_1} = \sum_{\nu>1}^{\infty} \frac{U_\nu^2}{U_1^2} \tag{6}$$

or by the flux distortion factor:

$$K_\Psi = \frac{2\Psi_L^2 - \Psi_1^2}{\Psi_1^2} = \sum_{\nu>1} \frac{\Psi_\nu^2}{\Psi_1^2}, \tag{7}$$

where (for m integer):

$$U_L = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} U_L^2 dW_1 t} \quad \text{— rms value of } u_L,$$

$$\Psi_L = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} \Psi_L^2 dW_1 t} \quad \text{— rms value of } \Psi_L, \tag{8}$$

and $\psi_L(t)$ is the integral value of $u_L(t)$; hence, $\Psi_1 = U_1/W_1$ and $\Psi_\nu = U_\nu/(w_1\nu)$. For an ohmistic load it is advisable to use (6); for an inductive load, (7), because in these cases these factors will be proportional to the current distortion factors:

$$K_i = K_u/R^2, \tag{9}$$

and

$$K_i = K_\Psi / L^2, \tag{10}$$

where R and L are the constant resistance or inductance of the load.

For bipolar modulation (*Fig. 2.a*), $U_L = U_{dc}$ and $U_1 = AU_{dc}$; therefore:

$$K_{ub} = \frac{2}{A^2} - 1. \tag{11}$$

For unipolar modulation, $U_L \leq U_{dc}$, the K_{uu} voltage distortion factor will therefore be less than for the bipolar one: $K_{uu} \leq K_{ub}$. The equality is according to the maximum value $U_1 = 4U_{dc}/\pi$:

$$K_{ub} = K_{uu} = \frac{2}{16}\pi^2 - 1 = 0.2337. \tag{12}$$

For unipolar modulation the voltage distortion can be written as:

$$K_{uu} = \frac{U_{dc}^2}{\pi U_1^2} \sum_i (\alpha_i - \alpha_{i-1}) - 1, \tag{13}$$

where the summation is distributed on all pulses in the region $0 < W_1 t \leq 2\pi$. But with a very good approximation

$$4\Psi_1 W_1 = 4U_1 \approx U_{dc} \sum_i (\alpha_i - \alpha_{i-1}), \tag{14}$$

since the right part of the equation gives the change of the flux for the period and $\Psi_{\max} \approx \Psi_1$. Given this, K_{uu} does not depend on m and (13) can be rewritten in the next term ($A \leq 1$):

$$K_{uu} = \frac{4}{\pi} \frac{U_{dc}}{U_1} - 1 \approx \frac{4}{\pi} \frac{1}{A} - 1. \tag{15}$$

K_{ub} and K_{uu} are drawn in *Fig. 5*.

The flux distortion factors (7) for the maximum value of ψ will be the same for both modulation strategies:

$$K_{\Psi b} = K_{\Psi u} = \frac{\pi^2}{12} \frac{\pi^2}{16} 2 - 1 = \frac{\pi^4}{96} - 1 = 0.01468,$$

where: $\Psi_L = \dfrac{\pi}{12} \dfrac{U_{dc}}{W_1}$ and $\Psi_1 = \dfrac{4}{\pi} \dfrac{U_{dc}}{W_1}$.

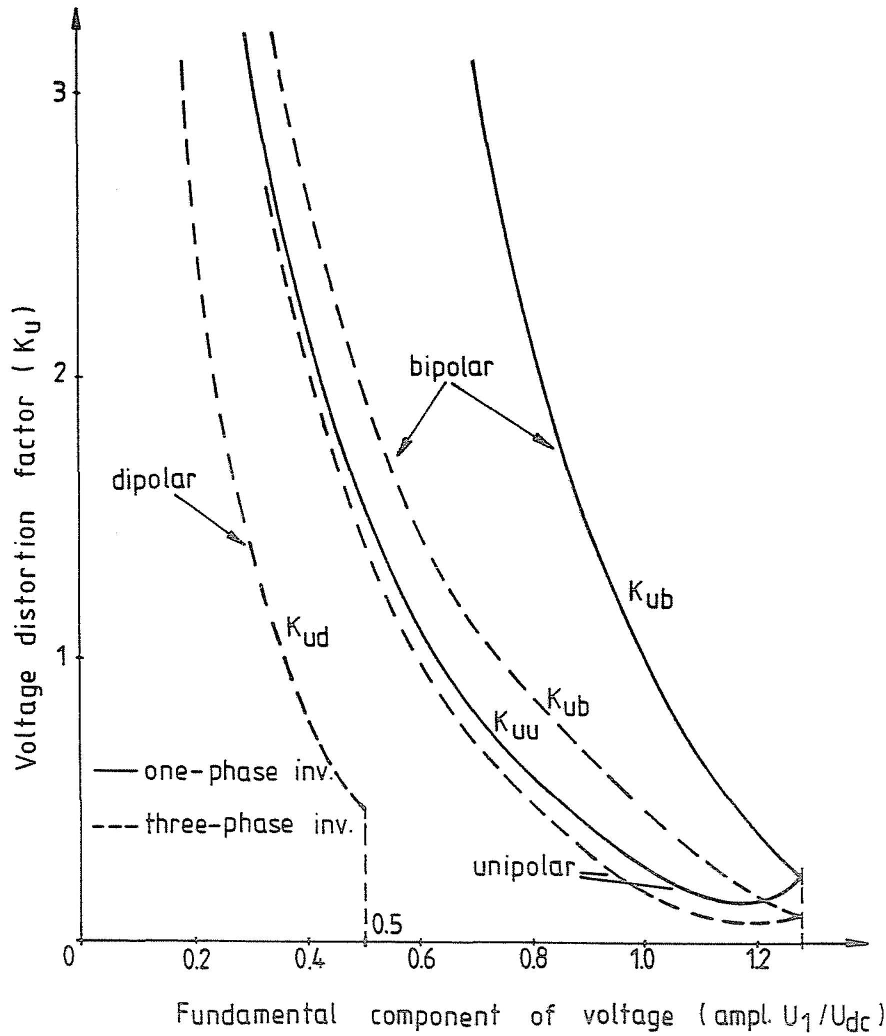


Fig. 5. Voltage distortion factor vs fundamental voltage

The analytical equations for $K_{\Psi b}$ and $K_{\Psi u}$ are used; the changes in these factors as functions of the fundamental voltage component are presented in *Fig. 6* and *Fig. 7* for the different values of m. In the case $U_1 \longrightarrow 0$, $K_{\Psi b}$ approaches the infinite value, the harmonics of the order Km being responsible for that, as $U_{Km}/U_1 \longrightarrow \infty$. Unipolar modulation eliminates these harmonics and therefore $K_{\Psi u}$ takes the final value at $U_1 \longrightarrow 0$. This value is determined by harmonics of order $2m \pm 1$, $4m \pm 1$, $6m \pm 1$, etc., for

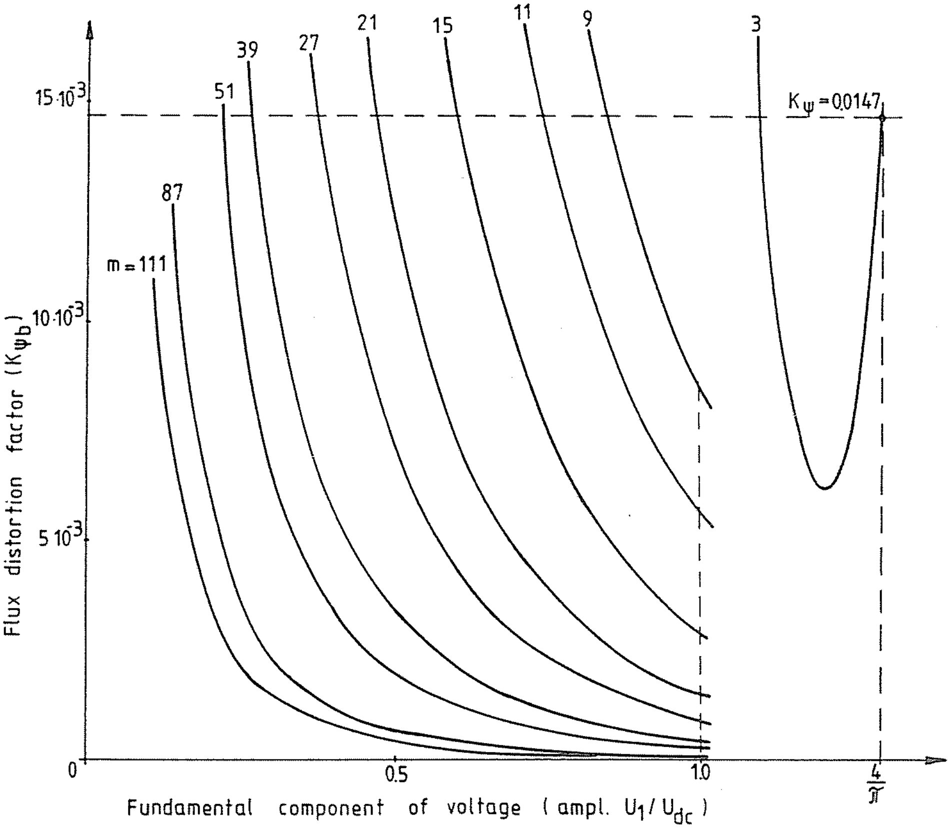


Fig. 6. Flux distortion factor vs fundamental voltage (one-phase bipolar PWM)

which $U_\nu/U_1 \longrightarrow 1$ if $U_1 \longrightarrow 0$ (for other harmonics $U_\nu/U_1 \longrightarrow 0$). Hence, the flux distortion factor for the unipolar modulation at $U_1 \longrightarrow 0$ can be written as follows:

$$K_{\Psi u\,\max} = \frac{1}{(2m-1)^2} + \frac{1}{(2m+1)^2} + \frac{1}{(4m-1)^2} + \frac{1}{(4m+1)^2} + \dots \quad (16)$$

or with the assumption, that $km \pm 1 \cong Km$:

$$K_{\Psi u\,\max} = \frac{2}{4m^2}\left[1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots\right] = \frac{\pi^2}{12m^2}. \quad (17)$$

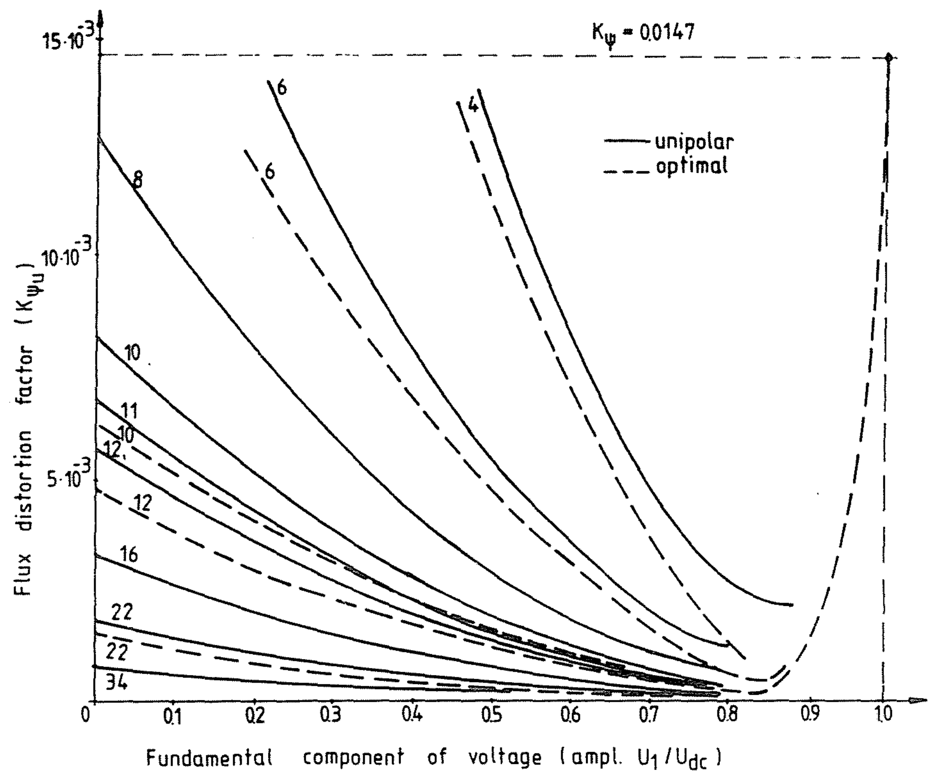


Fig. 7. Flux distortion factor vs fundamental voltage (one-phase unipolar PWM)

Comparing the bipolar and the unipolar modulations we can state the following:

a) In the region $0 \leq A \leq 1 (0 \leq U_1 \leq U_{dc})$ the values of K_u and K_Ψ monotonically decrease as the fundamental voltage increases.

b) The unipolar modulation strategy has a significant advantage compared with the bipolar one. It can be stated that the unipolar modulation in *Fig. 2.c* gives about the same result as the one in *Fig. 2.b*, but both modulation methods produce $K_{\Psi u}$ near the minimum possible value, which is obtained by the optimal distribution of pulses [4–6].

c) There is a reason to use inverters with the highest fundamental component about 70–80% from the possible maximum value of the component. With this, for a given commutation frequency of the inverter, we get the possibility of obtaining the minimum value of the distortion factors and, in parallel, the absence of low-order harmonics.

d) In the case of bipolar modulation, the order of the first significant harmonics is $m-2$, m and $m+2$. The frequency of the lowest one is

$$f_{m-2} = (m-2)f_1 = f_c - 2f_1, \tag{18}$$

where $f_c = mf_1$, the carrier frequency of the triangular wave. For unipolar modulation, the lowest order of the first important harmonics is $2m \pm 1$, $2m \pm 3$. The frequency of the $2m-1$ order is:

$$f_{2m-1} = (2m-1)f_1 = f_{cu} - f_1, \tag{19}$$

where $f_{cu} = 2mf_1$, the carrier frequency of the triangular wave. Hence, for the same number of pulses, the frequency of the lowest order harmonic is approximately the same.

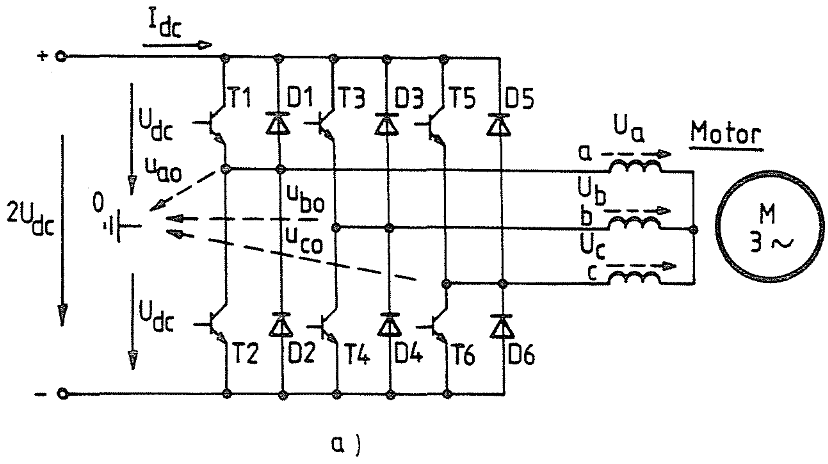
3. Three-phase Inverters

Three-phase inverters are presented in *Fig. 8*. For the conventional two-level inverters (*Fig. 8.a*) only bipolar modulation is possible, but for three-level inverters both bipolar and unipolar modulation methods are used. For three-phase systems, three symmetrically shifted sinusoidal reference waves are obtained, but the triangular carrier wave is the same in all phases.

a) Voltage Spectra

In three-wire three-phase systems all the zero sequence harmonics are canceled, hence, in (1) all harmonics with $n = 0, 3, 6, 9, \ldots$ (for example of the order m, $3m$, $9m$, etc.) are canceled. Therefore, only the harmonics of *Fig. 4* remain in the spectrum for bipolar modulation and from these only harmonics with continuous lines remain for the unipolar modulation.

For three-level inverters in [11] an interesting method of so-called bipolar modulation is suggested for the lower half region of the fundamental component. As presented in *Fig. 9*, for each phase two sinusoidal references are created with shifting ± 0.5 in relation to the middle point of the triangular wave. If the triangular wave is between two sinusoidal waves, the phase is connected to the middle point of the dc supply. In the opposite case, the phase is connected to $+U_{dc}$ (if the triangular wave is higher than the upper sinusoidal wave) or to $-U_{dc}$ (if the triangular wave is lower than the lower sinusoidal wave). In that case the amplitudes of all harmonics



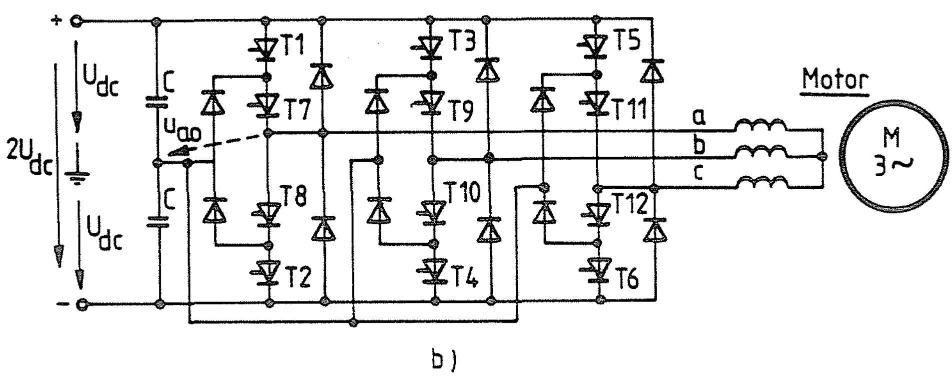


Fig. 8. Three-phase inverters
a) two-level
b) three-level

in *Fig. 4* must be multiplied by $\cos(\pi K/4)$, and therefore the harmonics with $K = 2, 6, 10, \ldots$ are eliminated. The order of the first important harmonics becomes $m \pm 2$, $4m \pm 1$, etc.; but in comparison with the bipolar modulation of *Fig. 2.a*, the number of the commutations will be twice as much.

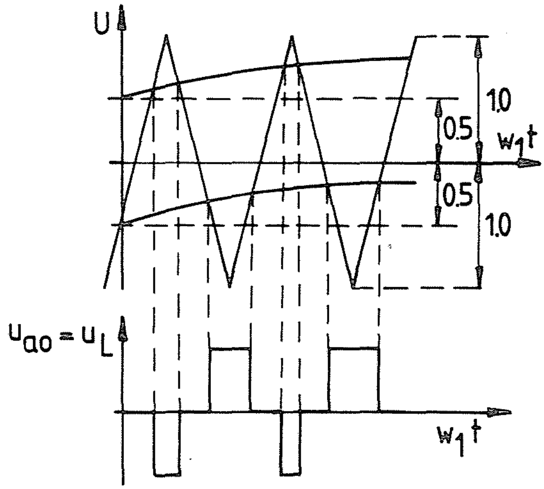


Fig. 9. Dipolar PWM

b) Distortion Factors

The voltage distortion factors are drawn in *Fig. 5*. For unipolar modulation K_{uu} decreases according to elimination of the harmonics with order $Km \pm 3$ and therefore this decrease is noticeable only for $A > 0.5$. In the case of bipolar modulation (in Park-vector notations) the next approximation equation is valid:

$$U_{RMS}^2 = \frac{1}{2\pi}\left(\frac{4}{3}U_{dc}\right)^2 \sum(\alpha_i - \alpha_{i-1}) \approx \frac{8}{\sqrt{3}\pi}U_{dc}U_1, \tag{20}$$

since for $A = U_1/U_{dc} < 1$:

$$\frac{1}{2\pi}\left(\frac{4}{3}U_{dc}\right)^2 \sum(\alpha_i - \alpha_{i-1}) \approx \frac{8}{\sqrt{3}\pi}\Psi_1 W_1 = \frac{8}{\sqrt{3}\pi}U_1. \tag{21}$$

With that for $A \leq 1$ we get:

$$K_{ub} = \frac{U_{RMS}^2}{U_1^2} - 1 = \frac{8}{\sqrt{3}\pi}\frac{U_{dc}}{U_1} - 1 = \frac{8}{\sqrt{3}\pi}\frac{1}{A} - 1. \tag{22}$$

From *Fig. 5* it can be seen that for three-phase systems the voltage distortion factors decrease considerably.

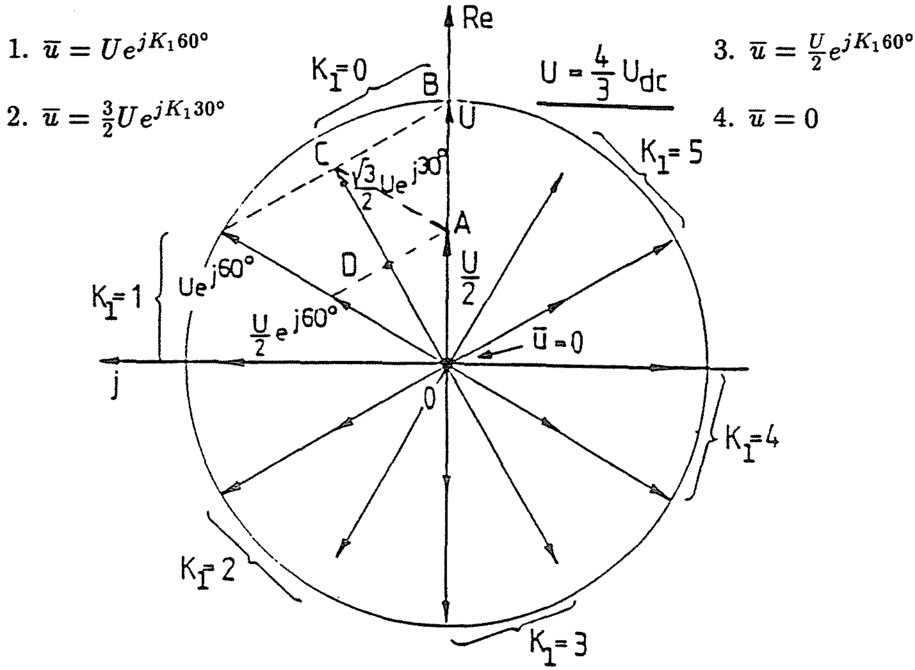


Fig. 10. Voltage-vectors of three-level inverters

For a maximum value of $U_1 = \frac{4}{\pi}U_{dc}$ the voltage distortion factor will be [12]

$$K_{uu} = K_{ub} = \left(\frac{4}{3}U_{dc}\right)^2\left(\frac{\pi}{4U_{dc}}\right)^2 - 1 = \frac{\pi^2}{9} - 1 = 0.09662. \tag{23}$$

In the case of bipolar modulation from the possible voltage vectors of *Fig. 10*, which can be obtained for three-phase inverters of *Fig. 8.b* [13], only vectors $\overline{u} = \frac{2}{3}U_{dc}e^{jK_1\pi}$ (where $K_1 = 0 \div 5$) and $\bar{u} = 0$ are used. Therefore (20) is valid, if $0 \leq A \leq 0.5$ and

$$K_{ud} = \frac{4}{\sqrt{3}\pi}\frac{1}{4} - 1. \tag{24}$$

This means that in *Fig. 5* the values of K_{ud} are derived by shifting each point of K_{ub} on the half values of U_1/U_{dc}.

The flux distortion factors for bipolar modulation are drawn in *Fig. 11* [13]. The interesting fact is that for a three-phase system the values of $K_{\Psi b}$ for $U_1 \longrightarrow 0$ are now determined by (17). Hence, for the same value of m,

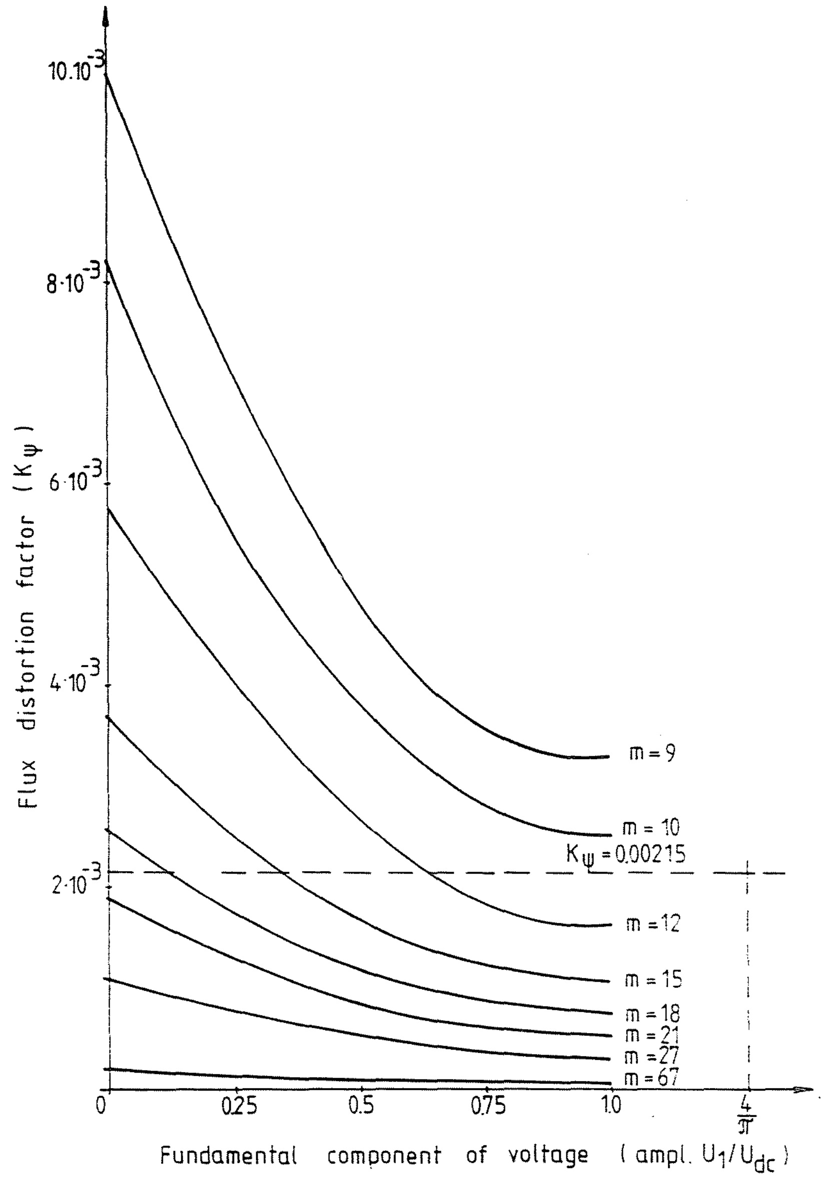


Fig. 11. Flux distortion factor vs. fundamental voltage (three-phase bipolar PWM)

the same flux distortion factors for $U_1 \longrightarrow 0$ are obtained for the bipolar and unipolar modulations.

The flux distortion factors in the case of the bipolar modulation change only in the region $0 \leq A \leq 0.9$ monotonically, in the region $0.9 \leq A \leq 1.0$ the flux distortion factor can increase for low value of m or stay about constant for $m > 21$. The next approximate equations can be used for $m > 9$:

$$\begin{aligned} A = 1 \qquad &\longrightarrow \qquad K_{\Psi b} = \frac{0.235}{m^2}, \\ A = 0.5 \qquad &\longrightarrow \qquad K_{\Psi b} = \frac{0.377}{m^2}, \\ A = 0 \qquad &\longrightarrow \qquad K_{\Psi b} = \frac{\pi^2}{12m^2}. \end{aligned} \tag{25}$$

For unipolar modulation the flux distortion factor for three-phase systems decreases very slightly, therefore, the curves of one-phase flux distortion factors of *Fig. 7* can be used. For $A \longrightarrow 0$, $K_{\Psi u}$ doesn't change; for $A = 1$, $K_{\Psi u}$ decreases by about $0.09/m^2$.

For bipolar modulation, the analogous equation to (17) can be derived for $U_1 \longrightarrow 0$:

$$K_{\Psi d} = \frac{1}{(4m-1)^2} + \frac{1}{(4m+1)^2} + \frac{1}{(8m+1)^2} + \frac{1}{(8m-1)^2} \approx \frac{\pi^2}{48m^2}. \tag{26}$$

The $K_\Psi(U_1)$ functions for $m = 9$, 15 and 27 are drawn in *Fig. 12*.

In *Fig. 12* the flux distortion factors are compared for the same commutation frequency of the three-level inverter semiconductors. If for unipolar modulation $m = 9$, for bipolar modulation $m = 9$ must be taken too (the number of the commutation decreases by half, but one commutation consists of turning-off and turning-on two-two semiconductors). The $U_1 \longrightarrow 0$ point will be the same, but for a high value of U_1 the difference in K_Ψ becomes significant. In *Fig. 12* the voltage spectra for $A = 1$ are also presented. Note that for two-level inverters $m = 9$ also gives the same commutation frequency of the semiconductors.

For region $0 \leq A \leq 0.5$ also in *Fig. 12*, the bipolar and one-side bipolar modulations are compared as well. For the bipolar modulation all the semiconductors of three-level inverters are used in the same manner; for one-side modulation this is possible only if we pass at least one time during each half period from the high to low sides (or reverse) of a three-level inverter. If for bipolar modulation we take $m = 9$, the same commutation frequency of semiconductors for the one-side bipolar modulation will be $m = 18$, and this commutation frequency will be the same as in the previous

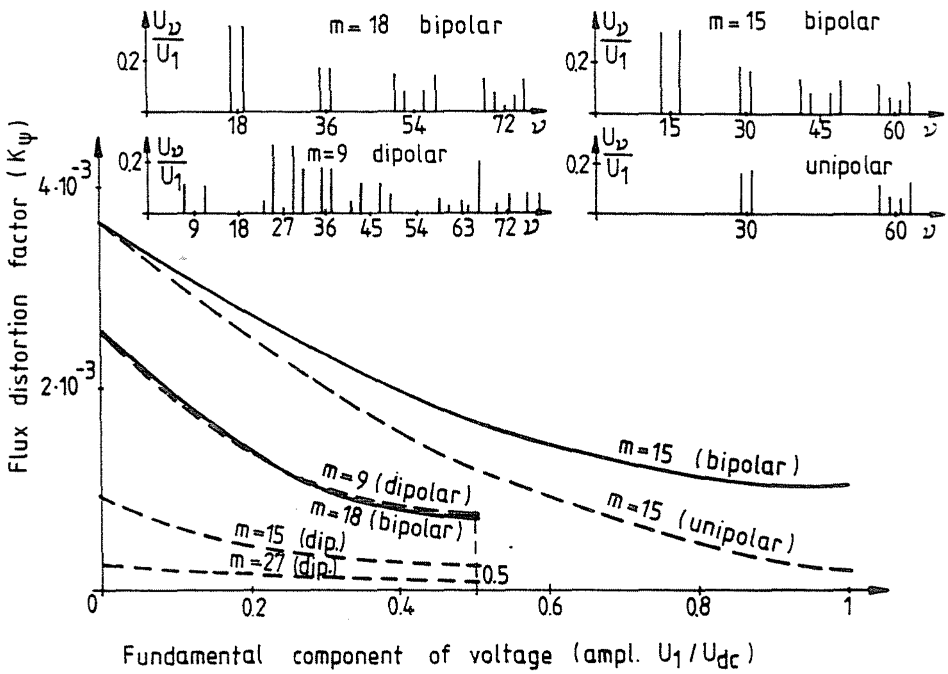


Fig. 12. Flux distortion factor vs. fundamental voltage (three-phase bipolar PWM and comparison of different PWM methods)

example. For $A \longrightarrow 0$ the same value of K_Ψ is obtained; for $A = 1$ we have $K_{\Psi d} = 7.9 \cdot 10^{-4}$ and $K_{\Psi b} = 7.8 \cdot 10^{-4}$. The voltage spectra for $A = 0.5$ are also presented in *Fig. 12*. Taking into account that for bipolar modulation we need a few additional commutations, we can state that two modulations produce the same result. But this result achieves a greater rate than what can be reached with the unipolar modulation.

Conclusion

The sinusoidal PWM methods offer a good opportunity for the realisation of inverter control. Unipolar modulation, for the same commutation frequency of transistors (GTOs), produces flux distortion factors (hence, the harmonic current load losses for inductive load) lower than bipolar modulation does, especially, for a high value of the fundamental voltage and one-phase applications. For three-level inverters, in the region $A < 0.5$, dipolar or one-side modulation can be used. Both methods produce much lower harmonic load losses than unipolar modulation does.

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